

# STANDARD COMPOSITION ALGEBRAS OF TYPE II

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**Abstract:** Among composition algebras, the well known ones are those with identity (so, finite dimensional, [2]), that is, Hurwitz algebras. These over a field of characteristic different from 2, by the generalized Hurwitz Theorem in [5], are isomorphic either to the base field, a separable quadratic extension of the base field, a generalized quaternion algebra or a generalized Cayley algebra.

A very complete study on composition algebras without identity, but satisfying an additional condition (either the associativeness of the norm, or the flexible identity, or a Moufang identity, or the third power associativeness, or the third and fourth powers associativeness, or degree two), has been presented in works due to Cuenca-Mira, Elduque, Myung, Okubo, Osborn, Pérez-Izquierdo and Sánchez-Campos. All the details can be found in [3] and references therein.

In this communication, over a field of characteristic different from two, we consider the standard composition algebras of type II associated to the Hurwitz algebras  $\mathbb{H}$  (quaternion algebra) and  $\mathbb{O}$  (octonion algebra). We focus on some identities satisfied by the former algebras and, concretely for the ones of level 2, we apply the random vectors method. See, for instance, [4] for more details about the mentioned process that involves Computational Linear Algebra on matrices.

Afterwards, we analyze if the obtained identities characterize the standard composition algebras of type II. On the one hand, we see that the level 2 identity that implies all level 2 identities of the standard composition algebra of type II associated to  $\mathbb{H}$ , as well as the corresponding one associated to  $\mathbb{O}$ , cannot accomplish that purpose. On the other hand, we conclude that if  $A$  is a composition algebra of arbitrary dimension, over a field of characteristic different from 2, that satisfies the identity  $x^2y = n(x)y$ , then  $A$  is standard of type II.

**keywords:** Hurwitz algebra; identity; standard composition algebra of type II.

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