

THE CLASSIFICATION OF NATURALLY REDUCTIVE HOMOGENEOUS SPACES IN DIMENSIONS $n \leq 6$

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Resumo: We present a new method for classifying naturally reductive homogeneous spaces – i.e. homogeneous Riemannian manifolds admitting a metric connection with skew torsion that has parallel torsion and curvature. This method is based on a deeper understanding of the holonomy algebra of connections with parallel skew torsion on Riemannian manifolds and the interplay of such connections with the geometric structure on the given Riemannian manifold. We reproduce by much easier arguments the known classifications in dimensions 3, 4 and 5, and obtain as a new result the complete classification in dimension 6. In each dimension, we also exhibit a ‘hierarchy’ of degeneracy for the torsion form, which we then treat case by case. For the complete degenerate cases, we prove results that are dimension independent. In some situations, we are able to show that any Riemannian manifold with parallel skew torsion has to be naturally reductive. We prove that a ‘generic’ parallel torsion form defines a quasi-Sasaki structure in dimension 5 and an almost complex structure in dimension 6.

palavras-chave: Naturally reductive spaces; connections with skew torsion; geometric structures.

Referências

- [1] I. Agricola, A.C. Ferreira, T. Friedrich: *The classification of naturally reductive homogeneous spaces in dimensions $n \leq 6$* . (To appear).