Proofs of $1 + 2 + \cdots + n = n(n+1)/2$

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Resumo: It is said that when Gauss was in primary school, his teacher asked him to add $1 + 2 + \cdots + 100$, and Gauss quickly answered $50 \times 101 = 5050$, by realising that $1 + 2 + \cdots + 100$ is the sum of the 50 terms $1 + 100, 2 + 99, \ldots, 50 + 51$, all equal to 101. Gauss's realisation gives rise to the nowadays well-known formula $1 + 2 + \cdots + n = n(n+1)/2$, which has many proofs.

In this talk we present some of the cutest proofs of Gauss's formula (and, when the proof allows it, a generalisation of Gauss's formula). For example, proofs by

- calculating the area of certain trapezoids;
- polynomial interpolation (and a generalisation to a formula for $1^p + 2^p + \cdots + n^p$);
- calculus of finite differences (and a generalisation to a formula for $1^{\underline{p}} + 2^{\underline{p}} + \cdots + n^{\underline{p}}$);
- counting the number of subsets of cardinality 2 of $\{0, 1, \ldots, n\}$ (and a generalisation to a formula for $\binom{1}{p} + \binom{2}{p} + \cdots + \binom{n}{p}$).

We keep this talk short, simple and sweet.

Palavras-chave: triangular number; triangle number.