Mathematical candies

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Resumo: When students express their feelings about mathematics, the usual adjectives are not "interesting", "beautiful", "pleasant" and "amusing", but "boring", "horrible", "painful" and "dull". This predisposition against mathematics is a wall that teachers of mathematics need to tear down to free the way for the learning of mathematics. How do we do this? By delighting students with "mathematical candies": little pieces of mathematics that are "interesting", "beautiful", "pleasant" and "amusing".

In this talk we present "mathematical candies" like the following ones.

- When Gauss's teacher asked him to add $1+2+\cdots+100$, he answered $50 \times 101 = 5050$ realising that $1 + 2 + \cdots + 100$ is the sum of the 50 terms $1 + 100, 2 + 99, \ldots, 50 + 51$ equal to 101. This generalises to $1+2+\cdots+n = n(n+1)/2$. We give a geometric proof of this formula.

- If we ask students to mention some irrationals numbers, we are already lucky if we hear $\sqrt{2}$, π and e. This is natural because almost all everyday numbers are rational, but deceiving because almost all real numbers are irrational. Can we show this to students? We give a elementary proof that at least half of the real numbers are irrational.

- Mathematicians can prove that an equation has a solution (1) constructively by giving a solution or (2) non-constructively without giving a solution. Most mathematicians accept both proofs, but a minority only accepts constructive proofs. We give an illustrative example of a simple equation with constructive and non-constructive proofs.

- Students are familiar with the classification of triangles as acute, right, obtuse, scalene, isosceles, equilateral, etc. But this "zoology" can easily become tedious. Can we fix this by doing something funny with it? We give a funny way of visualising all classes of triangles at once.

We keep this talk short, simple and sweet.

Palavras-chave: Mathematical candy; triangular/triangle number; irrational number; (non-)constructive proof; classification of triangles.